

# Pseudospin symmetry and its approximation in real nuclei

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The origin of pseudospin symmetry and its broken in real nuclei are discussed in the relativistic mean field theory. In the exact pseudospin symmetry, even the usual intruder orbits have degenerate partners. In real nuclei, pseudospin symmetry is approximate, and the partners of the usual intruder orbits will disappear. The difference is mainly due to the pseudo spin-orbit potential and the transition between them is discussed in details. The contribution of pseudospin-orbit potential for intruder orbits is quite large, compared with that for pseudospin doublets. The disappearance of the pseudospin partner for the intruder orbit can be understood from the properties of its wave function.

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Pseudospin symmetry was originally observed in spherical atomic nuclei more than 30 years ago [1,2], and later proved to be a good approximation in deformed nuclei [3], including triaxial deformed nuclei [4]. This symmetry has been used to explain features of deformed nuclei, including superdeformation [5], and identical bands [6–8] as well. The concept of pseudospin is based on experimental observations that single particle states with non-relativistic quantum numbers  $(n, l, j = l + 1/2)$  and  $(n - 1, l + 2, j = l + 3/2)$  lie very close in energy and can therefore be relabelled as pseudospin doublets:  $(\tilde{n} = n - 1, \tilde{l} = l + 1, \tilde{j} = \tilde{l} \pm 1/2)$ . For example, the pair of orbits  $(3s_{1/2}, 2d_{3/2})$  can be viewed as a pseudospin doublet  $(2\tilde{p}_{1/2}, 2\tilde{p}_{3/2})$ . While, all  $p_{1/2}$  states are pseudospin singlets  $\tilde{s}_{1/2}$ , and states with  $n = 1, j = l + 1/2$  are intruder orbital states without pseudospin partner.

Since the suggestion of pseudospin symmetry in atomic nuclei, there have been comprehensive efforts to understand its origin. Apart from the rather formal relabeling of quantum numbers, various proposals for an explicit transformation from the normal scheme to the pseudospin scheme have been made in the last twenty years, and the pseudospin symmetry is proved to be connected with the special ratio in the strengths of the spin-orbit and orbit-orbit interactions [9–12]. Despite the long history of pseudospin symmetry, the origin of symmetry has eluded explanation.

The relation between pseudospin symmetry and the

relativistic mean field (RMF) theory [13] was first noted in [10], in which Bahri et al. found that the RMF theory explains approximately the special ratio of the strengths of spin-orbit and orbit-orbit interactions in the non-relativistic calculations for exact pseudospin symmetry. No great progress was achieved until Ginocchio revealed that the pseudo-orbital angular momentum is nothing but the “orbital angular momentum” of the lower component of the Dirac spinor and built the connection between pseudospin symmetry and the equality in magnitude but difference in sign of the scalar potential  $V_s(r)$  and vector potential  $V_v(r)$  [14,15]. Based on Relativistic Continuum Hartree-Bogoliubov (RCHB) [16,17], it is shown that pseudospin symmetry is exact under a more general condition,  $d(V_s + V_v)/dr = 0$ , and the quality of the pseudospin approximation in real nuclei is connected with the competition between the pseudo-centrifugal barrier and the pseudospin-orbital potential [18,19].

Here, in this paper, we will show why the pseudospin symmetry is approximate and broken in real nuclei, and explain the partner of intruder orbit is missing. We will first solve Dirac equation for two kinds of harmonic oscillator potentials with exact spin symmetry and pseudospin symmetry, respectively. Taking  $^{208}\text{Pb}$  as an example, we will investigate how the pseudospin symmetry is a good approximation for normal orbits and why intruder orbit fails to have partner in real nuclei. A short summary is given at the end.

The Dirac equation of a nucleon with mass  $M$  moving in an attractive scalar potential  $V_s(\mathbf{r})$  and a repulsive vector potential  $V_v(\mathbf{r})$  can be written as,

$$[\boldsymbol{\alpha} \cdot \hat{\mathbf{p}} + \beta(M + V_s) + V_v] \Psi_i = E_i \Psi_i. \quad (1)$$

For spherical nuclei, the nucleon angular momentum  $\hat{\mathbf{J}}$ , and  $\hat{\kappa} = -\hat{\beta}(\hat{\sigma} \cdot \hat{\mathbf{L}} + 1)$  commute with the Dirac Hamiltonian, where  $\hat{\beta}$ ,  $\sigma$ , and  $\mathbf{L}$  are respectively the Dirac matrix, Pauli matrix, and orbital angular momentum. The eigenvalues of  $\hat{\kappa}$  are  $\kappa = \pm(j + 1/2)$  with  $-$  for aligned spin ( $s_{1/2}$ ,  $p_{3/2}$ , etc.) and  $+$  for unaligned spin ( $p_{1/2}$ ,  $d_{3/2}$ , etc.). For a given  $\kappa = \pm 1, \pm 2, \dots$ ,  $j = |\kappa| - \frac{1}{2}$ ,  $\ell = |\kappa + 1/2| - \frac{1}{2}$ ,  $\tilde{\ell} = |\kappa - 1/2| - 1/2$ . The wave functions can be classified according to their angular momentum  $j$ ,  $\kappa$ , and the radial quantum number  $n$ . In terms of the upper and lower radial functions  $F_{n\kappa}(r)$  and  $G_{n\kappa}(r)$ , Eq. (1), becomes [20]:

$$\left(\frac{d}{dr} + \frac{\kappa}{r}\right) F_{n\kappa}(r) = (M + E_{n\kappa} - \Delta) G_{n\kappa}(r), \quad (2)$$

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$$\left(\frac{d}{dr} - \frac{\kappa}{r}\right) G_{n\kappa}(r) = (M - E_{n\kappa} + V) F_{n\kappa}(r), \quad (3)$$

with  $\Delta = V_v - V_s$  and  $V = V_s + V_v$ . Eliminating  $G_{n\kappa}(r)$  or  $F_{n\kappa}(r)$  one can get the following Schrödinger-like equations,

$$\left[ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} - (M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + V) + \frac{d\Delta}{dr} \left( \frac{d}{dr} + \frac{\kappa}{r} \right) \right] F_{n\kappa}(r) = 0, \quad (4)$$

$$\left[ \frac{d^2}{dr^2} - \frac{\tilde{\ell}(\tilde{\ell}+1)}{r^2} - (M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + V) - \frac{dV}{dr} \left( \frac{d}{dr} - \frac{\kappa}{r} \right) \right] G_{n\kappa}(r) = 0, \quad (5)$$

and the same result can be obtained by solving either Eq.(4) or Eq.(5).

For  $\Delta = 0$  or  $\frac{d\Delta}{dr} = 0$ , Eq.(4) is reduced to

$$\left[ \frac{d^2}{dr^2} - \frac{\ell(\ell+1)}{r^2} \right] F_{n\kappa}(r) = [(M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + V)] F_{n\kappa}(r). \quad (6)$$

The eigen energies,  $E_{n\kappa}$ , depend only on  $n$  and  $\ell$ , i.e.,  $E_{n,\kappa} = E(n, \ell(\ell+1))$ . For  $\ell \neq 0$ , the states with  $j = \ell \pm 1/2$  are degenerate. This is a SU(2) spin symmetry.

For the Dirac equation with the following vector and scalar potentials,

$$V_v(r) = V_s(r) = \frac{1}{4} M \omega^2 r^2, \quad (7)$$

$\Delta = V_v(r) - V_s(r) = 0$ , Eq.(6) can be solved analytically with the eigen energy,

$$(E_{n\kappa} - M) \sqrt{\frac{E_{n\kappa} + M}{2M}} = \omega \left( 2n + \ell - \frac{1}{2} \right) \quad (n = 1, 2, \dots). \quad (8)$$

Now states with the same  $n$  and  $\ell$  will be degenerate. Of course as the potential is a special harmonic oscillator, the spectrum is more degenerated. The corresponding upper radial wavefunction is,

$$F_{n\kappa}(r) = \lambda_1 (\alpha r)^{\ell+1} e^{-\frac{1}{2} \alpha^2 r^2} L_{n-1}^{\ell+1/2}(\alpha^2 r^2), \quad (\alpha^2 = \omega \sqrt{M(E_{n\kappa} - M)}), \quad (9)$$

where  $\lambda_1$  is a normalization constant and  $L_{n-1}^{\ell+1/2}$  is the Laguerre polynomials. The lower radial wave function can be obtained from Eq.(2). For the spin partners, although the upper wave functions are the same, their lower wave functions are different because their  $\kappa$  are different.

In Fig. 1, the energy spectrum for the Dirac equation with  $V_v = V_s = 2.5 r^2$  is given by solving Eq.(6). Of course,  $\Delta = 0$  or  $\frac{d\Delta}{dr} = 0$  means the missing of the spin-orbital interaction, which cannot be true for real nuclei.

For  $V = 0$  or  $\frac{dV}{dr} = 0$ , Eq.(5) is reduced as

$$\left[ \frac{d^2}{dr^2} - \frac{\tilde{\ell}(\tilde{\ell}+1)}{r^2} \right] G_{n\kappa}(r) = [(M + E_{n\kappa} - \Delta)(M - E_{n\kappa} + V)] G_{n\kappa}(r). \quad (10)$$

The solution of the Dirac equation can be obtained from Eq.(10) and its eigen energies,  $E_{n\kappa}$ , depend only on  $n$  and  $\tilde{\ell}$ , i.e.,  $E_{n\kappa} = E(n, \tilde{\ell}(\tilde{\ell}+1))$ . For  $\tilde{\ell} \neq 0$ , the states with  $j = \tilde{\ell} \pm 1/2$  are degenerate. This is the pseudospin SU(2) symmetry. Except pseudospin singlet states with  $\tilde{\ell} = 0$ , every aligned state ( $j = \ell + 1/2 = \tilde{\ell} - 1/2$ ) will have a degenerate unaligned partner ( $j = \ell - 1/2 = \tilde{\ell} + 1/2$ ), and vice versa. Even the usual intruder orbital states may have partner now. These characters are very similar to the spin SU(2) symmetry.

For the Dirac equation with the following vector and scalar potentials,

$$V_v(r) = -V_s(r) = \frac{1}{4} M \omega^2 r^2, \quad (11)$$

$V = V_v(r) + V_s(r) = 0$ , Eq.(10) can be solved analytically with the eigen energy,

$$(E_{n\kappa} + M) \sqrt{\frac{E_{n\kappa} - M}{2M}} = \omega \left( 2n + \tilde{\ell} - \frac{1}{2} \right) \quad (n = 1, 2, \dots). \quad (12)$$

The states with the same  $n$  and  $\tilde{\ell}$  will be degenerate. The corresponding lower radial wavefunction is,

$$G_{n\kappa}(r) = \lambda_2 (\alpha r)^{\tilde{\ell}+1} e^{-\frac{1}{2} \alpha^2 r^2} L_{n-1}^{\tilde{\ell}+1/2}(\alpha^2 r^2) \quad (\alpha^2 = \omega \sqrt{M(E_{n\kappa} + M)}), \quad (13)$$

where  $\lambda_2$  is the normalization constant and  $L_{n-1}^{\tilde{\ell}+1/2}$  is the Laguerre polynomials. The upper radial wave function can be obtained from Eq.(3). For the pseudospin partners, although the lower wave functions are the same, their upper wave functions are different due to  $\kappa$ .

In Fig.2, the energy spectrum for the Dirac equation with  $V_v = -V_s = 2.5 r^2$  is given by solving Eq.(10). Similar to Fig.1, the spectrum is more degenerate due to the special harmonic oscillator used here. To make sure that the radial quantum number in Fig.2 is correct and even the intruder orbits have pseudospin partners, the spectrum in Fig.2 is confirmed by solving the Dirac equation (1) numerically as in Refs. [16,17].

Now the questions arise: in real nuclei why the pseudospin symmetry is broken and there is no partner for intruder orbit.

In real nuclei,  $\frac{dV}{dr} \neq 0$ , and pseudospin symmetry is only an approximation. The quality of the pseudospin symmetry approximation depends on the competition between the contributions of the pseudo-centrifugal potential,  $\frac{\tilde{\ell}(\tilde{\ell}+1)}{r^2}$ , and the pseudospin-orbit potential,  $\frac{dV}{dr} \frac{\kappa}{r(M-E+V)}$ , in Eq. (5).

In [18,19], Meng et al. analyzed the competition of the two potentials for pseudospin partners, and found that the contribution of the pseudospin-orbit potential is small, compared with that of the pseudo-centrifugal potential. Following the arguments and conjectures in Ref. [18,19], pseudospin symmetry is discussed for the nuclei with deformed shape or isospin asymmetry [21–23].

To understand why the intruder orbit has no partner is a challenging problem. Lots of works have been done along this line. In Ref. [24,25], the wave functions of the pseudospin partners and the structure of the radial nodes have been investigated. Here combining the energy spectrum in Fig.2 with that obtained from RCHB [16,17], we will try to understand why the intruder orbit has no partner.

In Fig.3, multiplied with the wave function  $G^2$ , the effective pseudospin-orbit potential  $\frac{\kappa}{r} \frac{dV}{dr}$  and the effective centrifugal barrier  $(M-E+V) \frac{\tilde{\ell}(\tilde{\ell}+1)}{r^2}$  in  $^{208}\text{Pb}$  have been compared in detail for the pseudospin singlets, the pseudospin doublets, and the intruder orbits.

For pseudospin singlets, we choose  $1p_{1/2}$  and  $2p_{1/2}$  as examples, and find that the pseudo-centrifugal potential is always zero. For the usual pseudospin doublets, the contribution of the effective pseudospin-orbit potential is much smaller than that of the effective pseudo-centrifugal potential, and the pseudospin symmetry is a good approximation. For the intruder orbits, the contribution of the effective pseudospin-orbit potential is comparable with and sometimes even larger than that of the effective pseudo-centrifugal potential. It can be seen that the pseudospin-orbit potential is responsible for the broken of pseudospin symmetry. So far it is not explained where the pseudospin partner of the intruder orbit in real nuclei has gone. In the following we will try to get the answer from the properties of the wave function.

For bound states, their solution can be obtained from

either one of the Schrödinger-like equations, Eqs.(4) and (5). Let  $F(r) = r^{-\kappa} f(r)$ , from Eq.(2) one get:

$$\frac{df}{dr} = r^{\kappa} (M + E_{n\kappa} - \Delta) G. \quad (14)$$

For  $r \sim 0$ , the solution of bound states for Eqs.(2) and (3) can be obtained analytically as:

$$F(r) = \lambda \text{Sign}(\kappa) \sqrt{\frac{E_{n\kappa} + M - \Delta}{2M + V - \Delta}} \sqrt{r} J_{|\kappa+1/2|}(kr) \quad (15)$$

$$G(r) = \lambda \sqrt{\frac{E_{n\kappa} - M - V}{2M + V - \Delta}} \sqrt{r} J_{|\kappa-1/2|}(kr), \quad (16)$$

where  $k = \sqrt{-(M+E-\Delta)(M-E+V)}$  and  $\lambda$  is a normalization factor (here assumed to be positive). For  $\kappa > 0$ , if the node number of lower wave function  $G$  is zero:  $n_G = 0$ , then  $G$  will be always positive. As  $r \sim 0$ , one has:

$$J_n(r) = r^n \left( \frac{2^{-n}}{\Gamma(1+n)} + O[r]^2 \right), \quad (17)$$

and depending on  $\kappa$ ,  $f(r)$  can be written as:

$$f(r) \sim \lambda \sqrt{\frac{E_{n\kappa} + M - \Delta}{2M + V - \Delta}} \frac{2^{-|\kappa+1/2|}}{\Gamma(1+|\kappa+1/2|)} \times \begin{cases} r^{2\kappa+1} & \kappa > 0 \\ -1 & \kappa < 0 \end{cases} \quad (18)$$

As the factor  $M + E_{n\kappa} - \Delta$  in Eq.(14) is always positive, for  $n_G = 0$  and  $\kappa > 0$ ,  $\frac{df}{dr}$  will be always positive, i.e.,  $f(r)$  increases with  $r$ . While for bound states, the wave function at large  $r$  should decrease as an exponential function of  $r$ , i.e.,  $f(\infty) = 0$ . Therefore for  $n_G = 0$  and  $\kappa > 0$ , the boundary condition of  $f$  for bound state can not be satisfied and there is no corresponding bound states. This means that the pseudospin partner of the intruder orbit will disappear. Similar conclusion has been drawn in Ref. [25,24]. To restore the pseudospin symmetry for intruder orbit, one possibility is that the factor  $(M + E_{n\kappa} - \Delta)$  can change sign with  $r$ , as the case in Fig.2.

In summary, the origin of pseudospin symmetry and its broken in real nuclei are discussed in the RMF theory. In the exact pseudospin symmetry, all the orbits except the singlets  $p_{1/2}$  have partners. In real nuclei, pseudospin symmetry is approximate, and the partners of the usual intruder orbits will disappear. The competition between the pseudo-centrifugal and pseudospin-orbit potentials decides the quality of pseudospin symmetry. The contribution of pseudospin-orbit potential for intruder orbits is quite large, compared with that for pseudospin doublets. The disappearance of the pseudospin partner for the intruder orbit can be understood from the properties of its wave function.

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FIG. 1. The eigen energy for the Dirac equation with  $V_v(r) = V_s(r) = \frac{1}{4}M\omega^2 r^2$ ,  $M = 10.0$ , and  $\omega = 1.0$ .

FIG. 2. Same as Fig.1 but for  $V_v(r) = -V_s(r) = \frac{1}{4}M\omega^2 r^2$ .

- (a) Pseudospin singlet states
- (b) Pseudospin doublet states
- (c) Intruder orbits

FIG. 3. The competition between the effective pseudo-centrifugal  $G^2(r)(E - M - V)\tilde{\ell}(\tilde{\ell} + 1)/r^2$  ( solid line ) and pseudospin-orbital potentials  $-G^2(r)\frac{dV}{dr}\frac{\kappa}{r}$  ( dashed line ).

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